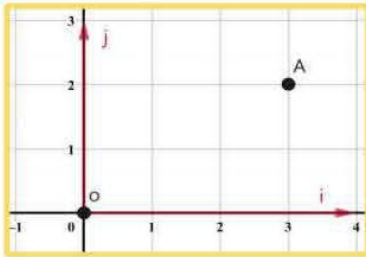


بنك الأسئلة لوحدة الأعداد العقدية



- ليكن لدينا العدد العقدي Z_A المبين بالشكل والمطلوب الإجابة عن الأسئلة التالية (من الرقم 1 إلى 4) :

1- يكون العدد العقدي Z_A بالشكل الديكارتي :

$Z_A = -3 + i2$	D	$Z_A = 3 - i2$	C	$Z_A = 2 + i3$	B	$Z_A = 3 + i2$	A
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$$Z_A = x + iy \Rightarrow x = 3, y = 2 \Rightarrow Z_A = 3 + i2$$

2- يكون مرافق العدد العقدي $\overline{Z_A}$:

$Z_A = -3 + i2$	D	$Z_A = 3 - i2$	C	$Z_A = 2 + i3$	B	$Z_A = 3 + i2$	A
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$$Z_A = x + iy \Rightarrow x = 3, y = 2 \Rightarrow \overline{Z_A} = x - iy \Rightarrow \overline{Z_A} = 3 - i2$$

3- تكون طويلة العدد العقدي Z_A :

$ Z_A = \sqrt{7}$	D	$ Z_A = 3$	C	$ Z_A = \sqrt{13}$	B	$ Z_A = \sqrt{12}$	A
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$$|Z_A| = \sqrt{x^2 + y^2} \Rightarrow |Z_A| = \sqrt{3^2 + 2^2} \Rightarrow |Z_A| = \sqrt{13}$$

4- يكون العدد العقدي $Z_{A1} = -2x - i3y$ يساوي :

$Z_{A1} = -6 - 6i$	D	$Z_{A1} = -5 - 5i$	C	$Z_{A1} = 5 + 5i$	B	$Z_{A1} = 6 - 6i$	A
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$$Z_{A1} = -2(3) - i(2)(3) \Rightarrow Z_{A1} = -6 - 6i$$

- ليكن لدينا الأعداد العقدية التالية : $Z_1 = 2 + i$, $Z_2 = -3 + 2i$, $Z_3 = 3i$ والمطلوب الإجابة عن الأسئلة التالية (من الرقم 5 إلى 13) :

5- يكون ناتج ما يلي : $Z_1 + Z_2 + \overline{Z_3}$

$+1 + 6i$	D	$+1$	C	$-1 + 6i$	B	-1	A
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$$Z_3 = 3i \Rightarrow \overline{Z_3} = -3i \Rightarrow Z_1 + Z_2 + \overline{Z_3} = 2 + i - 3 + 2i - 3i \Rightarrow Z_1 + Z_2 + \overline{Z_3} = -1$$

6- يكون ناتج ما يلي : $Z_1 - Z_2$

$1 + i$	D	$-5 + 3i$	C	$5 - i$	B	$-1 + 3i$	A
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$$Z_1 - Z_2 = 2 + i - (-3 + 2i) \Rightarrow Z_1 - Z_2 = 2 + i + 3 - 2i \Rightarrow Z_1 - Z_2 = 5 - i$$

7- يكون ناتج ما يلي : $Z_1 \cdot Z_2$

$1 + i$	D	$-8 + i$	C	$5 - i$	B	$-1 + 3i$	A
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$$Z_1 \cdot Z_2 = (2 + i) \cdot (-3 + 2i) \Rightarrow Z_1 \cdot Z_2 = -6 + 4i - 3i + 2i^2 \Rightarrow \\ \Rightarrow Z_1 \cdot Z_2 = -6 + 4i - 3i + 2(-1) \Rightarrow Z_1 \cdot Z_2 = -6 + 4i - 3i - 2 \Rightarrow Z_1 \cdot Z_2 = -8 + i$$

8- يكون ناتج ما يلي : Z_2^2

$5 - 12i$	D	$6 + 4i$	C	$13 + 12i$	B	$5 + 12i$	A
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$$Z_2^2 = (-3 + 2i)^2 \Rightarrow Z_2^2 = 9 - 12i + 4i^2 \Rightarrow Z_2^2 = 9 - 12i - 4 \Rightarrow Z_2^2 = 5 - 12i$$

9- يكون ناتج ما يلي : Z_3^3

$-9i$	D	27	C	$+9i$	B	$-27i$	A
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$$Z_3^3 = (3i)^3 \Rightarrow Z_3^3 = 27 i^2 \times i \Rightarrow Z_3^3 = -27i$$

10- تكون طولية العدد العقدي Z_1 :

$ Z_A = \sqrt{2}$	D	$ Z_1 = 5$	C	$ Z_1 = \sqrt{5}$	B	$ Z_1 = 2$	A
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$$|Z_1| = \sqrt{x^2 + y^2} \Rightarrow |Z_1| = \sqrt{2^2 + 1^2} \Rightarrow |Z_1| = \sqrt{5}$$

11- يكون ناتج ما يلي : $\frac{Z_1}{Z_2}$

$\frac{-4}{13} - \frac{i}{13}$	D	$\frac{-4}{13} - \frac{7i}{13}$	C	$\frac{-4}{5} - \frac{7i}{5}$	B	$\frac{-8}{13} - \frac{7i}{13}$	A
--------------------------------	---	---------------------------------	---	-------------------------------	---	---------------------------------	---

$$\frac{Z_1}{Z_2} = \frac{2+i}{-3+2i} \Rightarrow \frac{Z_1}{Z_2} = \frac{(2+i)(-3-2i)}{(-3+2i)(-3-2i)} \Rightarrow \frac{Z_1}{Z_2} = \frac{-6-4i-3i-2i^2}{9+4} \Rightarrow \frac{Z_1}{Z_2} = \frac{-4-7i}{13} \Rightarrow \frac{Z_1}{Z_2} = \frac{-4}{13} - \frac{7i}{13}$$

12- يكون ناتج ما يلي : $2Z_1 - 3\overline{Z_2}$

$10 + 6i$	D	$-2 - 2i$	C	$4 - i$	B	$10 - 2i$	A
-----------	---	-----------	---	---------	---	-----------	---

$$Z_2 = -3 + 2i \Rightarrow \overline{Z_2} = -3 - 2i \Rightarrow 2Z_1 - 3\overline{Z_2} = 2(2 + i) - 2(-3 - 2i) \\ \Rightarrow 2Z_1 - 3\overline{Z_2} = 4 + 2i + 6 + 4i \Rightarrow 2Z_1 - 3\overline{Z_2} = 10 + 6i$$

13- يكون ناتج ما يلي : $\frac{Z_2}{Z_3}$

$\frac{2}{3} - i$	D	$\frac{1}{3} - i$	C	$\frac{1}{3} + i$	B	$\frac{2}{3} + i$	A
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$$\frac{Z_2}{Z_3} = \frac{-3+2i}{3i} \Rightarrow \frac{Z_1}{Z_2} = \frac{(-3+2i)(-3i)}{(+3i)(-3i)} \Rightarrow \frac{Z_2}{Z_3} = \frac{9i-6i^2}{9} \Rightarrow \frac{Z_2}{Z_3} = \frac{6+9i}{9} \Rightarrow \frac{Z_2}{Z_3} = \frac{2}{3} + i$$

14- يكون ناتج ما يلي بالشكل الديكارتي : $z = \frac{\sqrt{3}+i}{2-2\sqrt{3}i}$

A $-\frac{1}{2}i$ B $+\frac{1}{2}i$ C $4\sqrt{3} + 4i$ D $4\sqrt{3} - i$

$$z = \frac{\sqrt{3}+i}{2-2\sqrt{3}i} \Rightarrow z = \frac{(\sqrt{3}+i)(2+2\sqrt{3}i)}{(2-2\sqrt{3}i)(2+2\sqrt{3}i)} \Rightarrow z = \frac{2\sqrt{3}+6i+2i+2\sqrt{3}i^2}{4+12} \Rightarrow z = \frac{2\sqrt{3}-2\sqrt{3}+8i}{16} \Rightarrow z = \frac{1}{2}i$$

15- يكون ناتج ما يلي بالشكل الديكارتي : $z = \frac{1}{2-i}$

A $\frac{2}{3} + \frac{1}{3}i$ B $\frac{2}{5} - \frac{1}{5}i$ C $\frac{2}{5} + \frac{1}{5}i$ D $\frac{2}{3} - \frac{1}{3}i$

$$z = \frac{1}{2-i} \Rightarrow z = \frac{(1)(2+i)}{(2-i)(2+i)} \Rightarrow z = \frac{2+i}{4+1} \Rightarrow z = \frac{2+i}{5} \Rightarrow z = \frac{2}{5} + \frac{1}{5}i$$

16- يكون ناتج ما يلي بالشكل الديكارتي : $z = \frac{1+i}{1-i}$

A 1 B $\frac{1}{2} + \frac{1}{2}i$ C $1 + i$ D i

$$z = \frac{1+i}{1-i} \Rightarrow z = \frac{(1+i)(1+i)}{(1-i)(1+i)} \Rightarrow z = \frac{1+2i+i^2}{1+1} \Rightarrow z = \frac{1+2i-1}{2} \Rightarrow z = \frac{2i}{2} \Rightarrow z = i$$

17- يكون ناتج ما يلي: $z = i^6$

A -1 B 1 C $-i$ D i

$$z = i^6 \Rightarrow z = i^4 \cdot i^2 \Rightarrow z = (1)(-1) \Rightarrow z = -1$$

18- يكون ناتج ما يلي: $z = i^8 + i^7$

A 1 B $1 - i$ C $-1 + i$ D i

$$z = i^8 + i^{12} \Rightarrow z = i^4(i^4 + i^3) \Rightarrow z = (1)((1) + (-i)) \Rightarrow z = 1 - i$$

19- يكون ناتج ما يلي: $z = i^{1000}$

A 1 B -1 C i D $-i$

$$z = (i^4)^{250} \Rightarrow z = (1)^{250} \Rightarrow z = 1$$

20- يكون ناتج ما يلي: $z = i^{25}$

A -1 B 1 C $-i$ D i

$$z = i^{25} \Rightarrow z = i^{24} \cdot i \Rightarrow z = (1)(i) \Rightarrow z = i$$

21- يكون ناتج ما يلي: $z = i^{15}$

A $-i$ B 1 C -1 D i

$$z = i^{15} \Rightarrow z = i^{12} \cdot i^3 \Rightarrow z = (1)(-i) \Rightarrow z = -i$$

22- يكون ناتج ما يلي: $z = (2i)^5$

A -32 B $32i$ C $16i$ D 16

$$z = (2i)^5 \Rightarrow z = 2^5 \cdot i^5 \Rightarrow z = (32)(i^4)(i) \Rightarrow z = (32)(1)(i) \Rightarrow z = 32i$$

- ليكن لدينا الأعداد العقدية التالية : $Z_1 = -2i$, $Z_2 = 2 - 2\sqrt{3}i$, $Z_3 = \sqrt{3} + i$ والمطلوب الإجابة عن الأسئلة التالية (من الرقم 23 إلى 25) :

23- يكون ناتج ما يلي : Z_1^3

8	D	$8i$	C	$-8i$	B	-8	A
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$$Z_1^3 = (-2i)^3 \Rightarrow Z_1^3 = (-2)^3 \cdot i^3 \Rightarrow Z_1^3 = (-8)(-i) \Rightarrow Z_1^3 = 8i$$

24- يكون ناتج ما يلي : Z_2^2

$-8 - 8\sqrt{3}i$	D	$16 - 8\sqrt{3}i$	C	$4 - 4\sqrt{3}i$	B	$8 + 8\sqrt{3}i$	A
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$$Z_2^2 = (2 - 2\sqrt{3}i)^2 \Rightarrow Z_2^2 = 4 - 8\sqrt{3}i + 12i^2 \Rightarrow Z_2^2 = -8 - 8\sqrt{3}i$$

25- يكون ناتج ما يلي : $Z_3 \cdot Z_2$

$2i$	D	$-2i$	C	$4\sqrt{3} + 4i$	B	$4\sqrt{3} - 4i$	A
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$$Z_3 \cdot Z_2 = (\sqrt{3} + i) \cdot (2 - 2\sqrt{3}i) \Rightarrow Z_3 \cdot Z_2 = 2\sqrt{3} - 6i + 2i - 2\sqrt{3}i^2 \Rightarrow \\ \Rightarrow Z_3 \cdot Z_2 = 2\sqrt{3} + 2\sqrt{3} - 6i + 2i \Rightarrow Z_3 \cdot Z_2 = 4\sqrt{3} - 4i$$

26- يكون العدد العقدي التالي بأبسط صورة بالشكل الديكارتي : $z = \frac{i+i^2}{2i}$

$\frac{1}{2} + i$	D	$\frac{1}{2} + \frac{1}{2}i$	C	$\frac{1}{2} - \frac{1}{2}i$	B	$1 + \frac{1}{2}i$	A
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$$z = \frac{i+i^2}{2i} \Rightarrow z = \frac{i(1+i)}{2i} \Rightarrow z = \frac{(1+i)}{2} \Rightarrow z = \frac{1}{2} + \frac{1}{2}i$$

27- يكون العدد العقدي التالي بأبسط صورة بالشكل الديكارتي : $z = \frac{i}{1+i} + \frac{3i}{1+i}$

$2 + 2i$	D	$\frac{1}{2} + \frac{1}{2}i$	C	$\frac{1}{2} - \frac{1}{2}i$	B	$2 - 2i$	A
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$$z = \frac{i}{1+i} + \frac{3i}{1+i} \Rightarrow z = \frac{4i}{1+i} \Rightarrow z = \frac{(4i)(1-i)}{(1+i)(1-i)} \Rightarrow z = \frac{4i-4i^2}{1+1} \Rightarrow z = \frac{4+4i}{2} \Rightarrow z = 2 + 2i$$

- ليكن لدينا الأعداد العقدية التالية : $Z_1 = 1 + i$, $Z_2 = 1 - 2i$ والمطلوب الإجابة عن الأسئلة التالية (من الرقم 28 إلى 30) :

28- يكون ناتج ما يلي : $|Z_1|$

$ Z_1 = 4$	D	$ Z_1 = \sqrt{2}$	C	$ Z_1 = 2$	B	$ Z_1 = 1$	A
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$$|Z_1| = \sqrt{x^2 + y^2} \Rightarrow |Z_1| = \sqrt{1^2 + 1^2} \Rightarrow |Z_1| = \sqrt{2}$$

29- يكون ناتج ما يلي : $|Z_1 + iZ_2|$

$\sqrt{5}$	D	13	C	$\sqrt{9}$	B	$\sqrt{13}$	A
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$$Z_1 + iZ_2 = 1 + i + i(1 - 2i) \Rightarrow Z_1 + iZ_2 = 1 + i + i - 2i^2 \Rightarrow Z_1 + iZ_2 = 3 + 2i \\ |Z_1 + iZ_2| = \sqrt{x^2 + y^2} \Rightarrow |Z_1 + iZ_2| = \sqrt{3^2 + 2^2} \Rightarrow |Z_1 + iZ_2| = \sqrt{13}$$

30- يكون ناتج ما يلي : $|Z_1^2 + iZ_2^2|$

A $\sqrt{5}$ B $\sqrt{17}$ C 17 D 5

$$Z_1^2 + iZ_2^2 = (1+i)^2 + i(1-2i)^2 = 1+2i+i^2 + i(1-4i-4i^2)$$

$$Z_1^2 + iZ_2^2 = 1+2i-1+i(1-4i-4) = 2i+i(-3-4i) = -i-4i^2$$

$$|Z_1^2 + iZ_2^2| = \sqrt{1^2 + 4^2} = \sqrt{1+16} \Rightarrow |Z_1^2 + iZ_2^2| = \sqrt{17}$$

31- الشكل المثلثي للعدد العقدي : $z = \sqrt{3} + i$

A $4 (\cos(\frac{\pi}{6}) + i \sin(\frac{\pi}{6}))$ B $2 (\cos(\frac{\pi}{3}) + i \sin(\frac{\pi}{3}))$ C $2 (\cos(\frac{\pi}{6}) + i \sin(\frac{\pi}{6}))$ D $4 (\cos(\frac{\pi}{3}) + i \sin(\frac{\pi}{3}))$

$$z = \sqrt{3} + i \Rightarrow r = \sqrt{\sqrt{3}^2 + 1^2} \Rightarrow r = \sqrt{3+1} \Rightarrow r = 2$$

$$\cos(\theta) = \frac{\sqrt{3}}{2}, \sin(\theta) = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{6} \Rightarrow z = 2 (\cos(\frac{\pi}{6}) + i \sin(\frac{\pi}{6}))$$

32- الشكل المثلثي للعدد العقدي : $z = -1 + \sqrt{3}i$

A $2 (\cos(\frac{\pi}{3}) + i \sin(\frac{\pi}{3}))$ B $4 (\cos(\frac{\pi}{3}) + i \sin(\frac{\pi}{3}))$ C $2 (\cos(\frac{2\pi}{3}) + i \sin(\frac{2\pi}{3}))$ D $2 (\cos(\frac{2\pi}{3}) + i \sin(\frac{2\pi}{3}))$

$$z = -1 + \sqrt{3}i \Rightarrow r = \sqrt{(-1)^2 + (\sqrt{3})^2} \Rightarrow r = \sqrt{1+3} \Rightarrow r = 2$$

$$\cos(\theta) = -\frac{1}{2}, \sin(\theta) = \frac{\sqrt{3}}{2} \Rightarrow \theta = \pi - \frac{\pi}{3} \Rightarrow \theta = \frac{2\pi}{3} \Rightarrow z = 2 (\cos(\frac{2\pi}{3}) + i \sin(\frac{2\pi}{3}))$$

33- الشكل المثلثي للعدد العقدي : $z = 2$

A $2 (\cos(0) + i \sin(0))$ B $4 (\cos(0) + i \sin(0))$ C $4 (\cos(\frac{\pi}{2}) + i \sin(\frac{\pi}{2}))$ D $2 (\cos(\pi) + i \sin(\pi))$

$$z = 2 \Rightarrow r = \sqrt{2^2} \Rightarrow r = 2$$

$$\cos(\theta) = \frac{2}{2} = 1, \sin(\theta) = \frac{0}{2} = 0 \Rightarrow \theta = 0 \Rightarrow z = 2 (\cos(0) + i \sin(0))$$

34- الشكل المثلثي للعدد العقدي : $z = -3$

A $9 (\cos(\pi) + i \sin(\pi))$ B $3 (\cos(0) + i \sin(0))$ C $3 (\cos(\frac{\pi}{2}) + i \sin(\frac{\pi}{2}))$ D $3 (\cos(\pi) + i \sin(\pi))$

$$z = -3 \Rightarrow r = 3 \Rightarrow \theta = \pi \Rightarrow z = 3 (\cos(\pi) + i \sin(\pi))$$

35- الشكل المثلثي للعدد العقدي : $z = i$

A $[1, \frac{\pi}{2}]$ B $[1, 0]$ C $[1, \pi]$ D $[1, \frac{3\pi}{2}]$

$$z = i \Rightarrow r = 1 \Rightarrow \theta = \frac{\pi}{2} \Rightarrow z = 1 (\cos(\frac{\pi}{2}) + i \sin(\frac{\pi}{2})) \Rightarrow z = [1, \frac{\pi}{2}]$$

36- الشكل المثلثي للعدد العقدي : $z = -4i$

A $[4, \frac{\pi}{2}]$ B $[4, \frac{3\pi}{2}]$ C $[4, \pi]$ D $[4, 0]$

$$z = -4i \Rightarrow r = 4 \Rightarrow \theta = -\frac{\pi}{2} \Rightarrow \theta = \frac{3\pi}{2} \Rightarrow z = 4 (\cos(\frac{3\pi}{2}) + i \sin(\frac{3\pi}{2})) \Rightarrow z = [4, \frac{3\pi}{2}]$$

-37 الشكل الديكارتي للعدد العقدي : $z = [\sqrt{2}, \frac{\pi}{4}]$

$z = \frac{1}{2} + \frac{1}{2}i$	D	$z = 1 + i$	C	$z = \sqrt{2} + \sqrt{2}i$	B	$z = 2 + 2i$	A
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$$z = [\sqrt{2}, \frac{\pi}{4}] \Rightarrow z = \sqrt{2} (\cos(\frac{\pi}{4}) + i \sin(\frac{\pi}{4})) \Rightarrow z = \sqrt{2} (\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i)$$

$$\Rightarrow z = \frac{\sqrt{2}}{\sqrt{2}} + \frac{\sqrt{2}}{\sqrt{2}}i \Rightarrow z = 1 + i$$

-38 الشكل الديكارتي للعدد العقدي : $z = [5, \pi]$

$z = -5$	D	$z = 5i$	C	$z = 5$	B	$z = -5i$	A
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$$z = [5, \pi] \Rightarrow z = 5 (\cos(\pi) + i \sin(\pi)) \Rightarrow z = 5(-1 + 0i) \Rightarrow z = -5$$

-39 الشكل الديكارتي للعدد العقدي : $z = [2, \frac{5\pi}{3}]$

$z = -1 - i$	D	$z = -1 - \sqrt{3}i$	C	$z = 1 - i$	B	$z = 1 - \sqrt{3}i$	A
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$$z = [2, \frac{5\pi}{3}] \Rightarrow z = 2 (\cos(\frac{5\pi}{3}) + i \sin(\frac{5\pi}{3})) \Rightarrow z = 2 (\frac{1}{2} - \frac{\sqrt{3}}{2}i)$$

$$\Rightarrow z = \frac{2}{2} - \frac{2\sqrt{3}}{2}i \Rightarrow z = 1 - \sqrt{3}i$$

-40 ليكن لدينا الأعداد العقدية التالية : $Z_1 = 1 + i$, $Z_2 = 1 + \sqrt{3}i$ يكون $Z_1 \cdot Z_2$ بالشكل المثلثي :

$[2, \frac{7\pi}{12}]$	D	$[2\sqrt{2}, \frac{\pi}{12}]$	C	$[2\sqrt{2}, \frac{7\pi}{12}]$	B	$[2\sqrt{2}, \frac{5\pi}{12}]$	A
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$$Z_1 = 1 + i \Rightarrow Z_1 = r = \sqrt{1^2 + 1^2} \Rightarrow r = \sqrt{2} \Rightarrow \cos(\theta) = \frac{1}{\sqrt{2}}, \sin(\theta) = \frac{1}{\sqrt{2}} \Rightarrow \theta = \frac{\pi}{4} \Rightarrow z = [\sqrt{2}, \frac{\pi}{4}]$$

$$Z_2 = 1 + \sqrt{3}i \Rightarrow r = \sqrt{1^2 + (\sqrt{3})^2} = \sqrt{1+3} \Rightarrow r = 2 \Rightarrow \cos(\theta) = \frac{1}{2}, \sin(\theta) = \frac{\sqrt{3}}{2} \Rightarrow \theta = \frac{\pi}{3} \Rightarrow [2, \frac{\pi}{3}]$$

$$Z_1 \cdot Z_2 = [\sqrt{2}, \frac{\pi}{4}] \cdot [2, \frac{\pi}{3}] = [2\sqrt{2}, \frac{\pi}{4} + \frac{\pi}{3}] = [2\sqrt{2}, \frac{3\pi}{12} + \frac{4\pi}{12}] \Rightarrow Z_1 \cdot Z_2 = [2\sqrt{2}, \frac{7\pi}{12}]$$

-41 يعطى العدد العقدي التالي بالشكل الديكارتي : $z = \frac{1+\sqrt{3}i}{1-\sqrt{3}i}$

$z = \frac{1}{2} + \frac{\sqrt{3}}{2}i$	D	$z = \frac{-1}{2} + \frac{\sqrt{3}}{2}i$	C	$z = \frac{\sqrt{3}}{2} + \frac{1}{2}i$	B	$z = \frac{1}{2} - \frac{\sqrt{3}}{2}i$	A
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$$z = \frac{1+\sqrt{3}i}{1-\sqrt{3}i} \Rightarrow z = \frac{(1+\sqrt{3}i)(1+\sqrt{3}i)}{(1-\sqrt{3}i)(1+\sqrt{3}i)} \Rightarrow z = \frac{1+2\sqrt{3}i+3i^2}{1-3i^2} \Rightarrow z = \frac{1+2\sqrt{3}i-3}{4} \Rightarrow z = \frac{-1+2\sqrt{3}i}{4} \Rightarrow z = \frac{-1}{2} + \frac{\sqrt{3}}{2}i$$

-42 ليكن لدينا العدد العقدي التالي : $z = \frac{-1}{2} + \frac{\sqrt{3}}{2}i$ يكون ناتج z^3 بالشكل الديكارتي :

$z^3 = 1$	D	$z^3 = -i$	C	$z^3 = i$	B	$z^3 = -1$	A
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$$z = \frac{-1}{2} + \frac{\sqrt{3}}{2}i \Rightarrow r = \sqrt{(\frac{1}{2})^2 + (\frac{\sqrt{3}}{2})^2} \Rightarrow r = \sqrt{\frac{1}{4} + \frac{3}{4}} \Rightarrow r = \sqrt{\frac{4}{4}} \Rightarrow r = \sqrt{1} \Rightarrow r = 1$$

$$\cos(\theta) = \frac{-1}{2} = -\frac{1}{2}, \sin(\theta) = \frac{\sqrt{3}}{2} \Rightarrow \theta = \pi - \frac{\pi}{3} = \frac{3\pi}{3} - \frac{\pi}{3} \Rightarrow \theta = \frac{2\pi}{3} \Rightarrow z = [1, \frac{2\pi}{3}]$$

$$z^3 = [1, \frac{2\pi}{3}]^3 \Rightarrow z^3 = [1^3, 3 \times \frac{2\pi}{3}] = [1, 2\pi] = 1(\cos(2\pi) + i \sin(2\pi)) = 1(1 + 0i) \Rightarrow z^3 = 1$$

43- ليكن لدينا العدد العقدي التالي : $z = \frac{\sqrt{3}}{2} + \frac{1}{2}i$ يكون ناتج z^{16} بالشكل المثلثي :

$[2, \frac{\pi}{6}]$	D	$[1, \frac{\pi}{6}]$	C	$[2, \frac{8\pi}{3}]$	B	$[1, \frac{8\pi}{3}]$	A
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$$z = \frac{\sqrt{3}}{2} + \frac{1}{2}i \Rightarrow r = \sqrt{(\frac{1}{2})^2 + (\frac{\sqrt{3}}{2})^2} \Rightarrow r = \sqrt{\frac{1}{4} + \frac{3}{4}} \Rightarrow r = \sqrt{\frac{4}{4}} \Rightarrow r = \sqrt{1} \Rightarrow r = 1$$

$$\cos(\theta) = \frac{\frac{\sqrt{3}}{2}}{1} = \frac{\sqrt{3}}{2}, \sin(\theta) = \frac{\frac{1}{2}}{1} = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{6} \Rightarrow z = [1, \frac{\pi}{6}] \Rightarrow$$

$$z^{16} = [1, \frac{\pi}{6}]^{16} \Rightarrow z^{16} = [1^{16}, 16 \times \frac{\pi}{6}] \Rightarrow z^{16} = [1, \frac{8\pi}{3}]$$

44- ليكن لدينا العدد العقدي التالي : $z = \frac{1+\sqrt{3}i}{\sqrt{3}+i}$ يكون ناتج z^{24} بالشكل المثلثي :

$z^{24} = -i$	D	$z^{24} = i$	C	$z^{24} = 1$	B	$z^{24} = -1$	A
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$$Z_1 = 1 + \sqrt{3}i \Rightarrow r = \sqrt{\sqrt{3}^2 + 1^2} = \sqrt{3+1} \Rightarrow r = 2 \Rightarrow \cos(\theta) = \frac{1}{2}, \sin(\theta) = \frac{\sqrt{3}}{2} \Rightarrow \theta = \frac{\pi}{3} \Rightarrow [2, \frac{\pi}{3}]$$

$$z_2 = \sqrt{3} + i \Rightarrow r = \sqrt{\sqrt{3}^2 + 1^2} = \sqrt{3+1} \Rightarrow r = 2 \Rightarrow \cos(\theta) = \frac{\sqrt{3}}{2}, \sin(\theta) = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{6} \Rightarrow [2, \frac{\pi}{6}]$$

$$z = \frac{z_1}{z_2} = [\frac{2}{2}, \frac{\pi}{3} - \frac{\pi}{6}] = [\frac{2}{2}, \frac{2\pi}{6} - \frac{\pi}{6}] \Rightarrow z = [1, \frac{\pi}{6}] \Rightarrow z^{24} = [1, \frac{\pi}{6}]^{24} = [1^{24}, 24 \times \frac{\pi}{6}]$$

$$z^{24} = [1, 4\pi] \Rightarrow z^{24} = 1(\cos(4\pi) + i\sin(4\pi)) = 1(1 + 0i) \Rightarrow z^{24} = 1$$

• ليكن لدينا الأعداد العقدية التالية : $Z = \frac{1-3i}{1+i} + 2i + \frac{3}{2}\sqrt{3}i - \frac{1}{2}$ والمطلوب الإجابة عن الأسئلة التالية (من الرقم 45 إلى 48) :

45- يكون العدد العقدي Z بالشكل $z = x + iy$

$z = -1 + \frac{3}{2}\sqrt{3}i$	D	$z = \frac{3}{2} + \frac{3}{2}\sqrt{3}i$	C	$z = 1 + \frac{3}{2}\sqrt{3}i$	B	$z = -\frac{3}{2} + \frac{3}{2}\sqrt{3}i$	A
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$$Z = \frac{1-3i}{1+i} + 2i + \frac{3}{2}\sqrt{3}i - \frac{1}{2} = \frac{(1-3i)(1-i)}{(1+i)(1-i)} + 2i + \frac{3}{2}\sqrt{3}i - \frac{1}{2} = \frac{1-i-3i+3i^2}{1+1} + 2i + \frac{3}{2}\sqrt{3}i - \frac{1}{2}$$

$$z = \frac{-2-4i}{2} + 2i + \frac{3}{2}\sqrt{3}i - \frac{1}{2} = -1 - 2i + 2i + \frac{3}{2}\sqrt{3}i - \frac{1}{2} = \frac{-2}{2} - \frac{1}{2} + \frac{\sqrt{3}}{2}i \Rightarrow z = \frac{-3}{2} + \frac{3}{2}\sqrt{3}i$$

46- يكون العدد العقدي Z بالشكل $z = r(\cos(\theta) + i\sin(\theta))$

$3(\cos(\frac{2\pi}{3}) + i\sin(\frac{2\pi}{3}))$	D	$9(\cos(\frac{\pi}{3}) + i\sin(\frac{\pi}{3}))$	C	$3(\cos(\frac{\pi}{6}) + i\sin(\frac{\pi}{6}))$	B	$9(\cos(\frac{\pi}{6}) + i\sin(\frac{\pi}{6}))$	A
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$$z = \frac{3}{2} + \frac{3}{2}\sqrt{3}i \Rightarrow r = \sqrt{(\frac{3}{2})^2 + (\frac{3}{2}\sqrt{3})^2} = \sqrt{\frac{9}{4} + \frac{27}{4}} = \sqrt{\frac{36}{4}} = \sqrt{9} \Rightarrow r = 3$$

$$\cos(\theta) = \frac{\frac{3}{2}}{3} = -\frac{1}{2}, \sin(\theta) = \frac{\frac{3}{2}\sqrt{3}}{3} = \frac{\sqrt{3}}{2} \Rightarrow \theta = \pi - \frac{\pi}{3} = \frac{2\pi}{3} \Rightarrow z = 3(\cos(\frac{2\pi}{3}) + i\sin(\frac{2\pi}{3}))$$

47- يكون ناتج ما يلي : z^3

$z^3 = 27i$	D	$z^3 = -9i$	C	$z^3 = 9$	B	$z^3 = 27$	A
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$$z = [3, \frac{2\pi}{3}] \Rightarrow z^3 = [3, \frac{2\pi}{3}]^3 \Rightarrow z^3 = [27, 2\pi] \Rightarrow z^3 = 27(\cos(2\pi) + i\sin(2\pi)) \Rightarrow z^3 = 27(1 + i0) = 27$$

-48 ليكن لدينا الأعداد العقدية التالية : $Z = \frac{\sqrt{3}+i}{\sqrt{3}-i} + \frac{\sqrt{3}-i}{\sqrt{3}+i} + i - 1$ يكون ناتج $z + \frac{1}{z}$ بالشكل الديكارتي :

$z = i$ **D** $z = i + 1$ **C** $z = 0$ **B** $z = 2i$ **A**

$$Z = \frac{(\sqrt{3}+i)(\sqrt{3}+i)}{(\sqrt{3}-i)(\sqrt{3}+i)} + \frac{(\sqrt{3}-i)(\sqrt{3}-i)}{(\sqrt{3}+i)(\sqrt{3}-i)} + i - 1 = \frac{3+2\sqrt{3}i+i^2}{3+1} + \frac{3-2\sqrt{3}i+i^2}{3+1} + i - 1 = \frac{3+3-1-1}{4} + i - 1$$

$$Z = \frac{4}{4} + i - 1 = 1 - 1 + i = i \Rightarrow z + \frac{1}{z} = i + \frac{1}{i} = i + \frac{i}{i^2} \Rightarrow Z = i + \frac{i}{-1} \Rightarrow Z = i - i \Rightarrow Z = 0$$

-49 الشكل المثلثي للعدد العقدي : $z = -\sqrt{6} + \sqrt{2}i$

$z = [2\sqrt{2}, \frac{\pi}{3}]$ **D** $z = [2\sqrt{2}, \frac{5\pi}{6}]$ **C** $z = [2\sqrt{2}, \frac{2\pi}{3}]$ **B** $z = [2\sqrt{2}, \frac{\pi}{6}]$ **A**

$$z = -\sqrt{6} + \sqrt{2}i \Rightarrow r = \sqrt{\sqrt{6}^2 + \sqrt{2}^2} = \sqrt{6+2} = \sqrt{8} = \sqrt{4 \times 2} \Rightarrow r = 2\sqrt{2}$$

$$\cos(\theta) = \frac{-\sqrt{6}}{2\sqrt{2}} = \frac{-\sqrt{3} \times \sqrt{2}}{2\sqrt{2}} = \frac{-\sqrt{3}}{2}, \sin(\theta) = \frac{\sqrt{2}}{2\sqrt{2}} = \frac{1}{2} \Rightarrow \theta = \pi - \frac{\pi}{6} \Rightarrow \theta = \frac{5\pi}{6} \Rightarrow z = [2\sqrt{2}, \frac{5\pi}{6}]$$

-50 الشكل المثلثي للعدد العقدي : $z = -\sqrt{5} - \sqrt{15}i$

$z = [2\sqrt{5}, \frac{4\pi}{3}]$ **D** $z = [5\sqrt{2}, \frac{\pi}{6}]$ **C** $z = [2\sqrt{5}, \frac{2\pi}{3}]$ **B** $z = [2\sqrt{5}, \frac{\pi}{6}]$ **A**

$$z = -\sqrt{5} - \sqrt{15}i \Rightarrow r = \sqrt{\sqrt{5}^2 + \sqrt{15}^2} = \sqrt{5+15} = \sqrt{20} = \sqrt{4 \times 5} \Rightarrow r = 2\sqrt{5}$$

$$\cos(\theta) = \frac{-\sqrt{5}}{2\sqrt{5}} = \frac{-1}{2}, \sin(\theta) = \frac{-\sqrt{15}}{2\sqrt{5}} = \frac{-\sqrt{3} \times \sqrt{5}}{2\sqrt{5}} = \frac{-\sqrt{3}}{2} \Rightarrow \theta = \pi + \frac{\pi}{3} \Rightarrow \theta = \frac{4\pi}{3} \Rightarrow z = [2\sqrt{5}, \frac{4\pi}{3}]$$

-51 الشكل المثلثي للعدد العقدي : $z = 3 - 3i$

$z = [3, \frac{7\pi}{4}]$ **D** $z = [9, \frac{\pi}{4}]$ **C** $z = [3\sqrt{2}, \frac{\pi}{4}]$ **B** $z = [3\sqrt{2}, \frac{7\pi}{4}]$ **A**

$$z = 3 - 3i \Rightarrow r = \sqrt{3^2 + 3^2} = \sqrt{9+9} = \sqrt{18} = \sqrt{9 \times 2} \Rightarrow r = 3\sqrt{2}$$

$$\cos(\theta) = \frac{3}{3\sqrt{2}} = \frac{1}{\sqrt{2}}, \sin(\theta) = \frac{-3}{3\sqrt{2}} = \frac{-1}{\sqrt{2}} \Rightarrow \theta = 2\pi - \frac{\pi}{4} \Rightarrow \theta = \frac{7\pi}{4} \Rightarrow z = [3\sqrt{2}, \frac{7\pi}{4}]$$

-52 الشكل المثلثي للعدد العقدي : $z = \frac{\sqrt{6}}{1+i}$

$z = [3, \frac{7\pi}{4}]$ **D** $z = [\sqrt{2}, \frac{\pi}{4}]$ **C** $z = [\sqrt{3}, \frac{7\pi}{4}]$ **B** $z = [\sqrt{2}, \frac{\pi}{4}]$ **A**

$$z = \frac{\sqrt{6}}{1+i} = \frac{[\sqrt{6}, 0]}{\sqrt{2}(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i)} = \frac{[\sqrt{6}, 0]}{[\sqrt{2}, \frac{\pi}{4}]} = [\frac{\sqrt{6}}{\sqrt{2}}, 0 - \frac{\pi}{4}] = [\frac{\sqrt{3}\sqrt{2}}{\sqrt{2}}, 2\pi - \frac{\pi}{4}] \Rightarrow z = [\sqrt{3}, \frac{7\pi}{4}]$$

-53 الشكل المثلثي للعدد العقدي : $z = \frac{i}{2\sqrt{3}+2i}$

$z = [\frac{1}{2}, \frac{\pi}{6}]$ **D** $z = [\frac{1}{4}, \frac{\pi}{3}]$ **C** $z = [\frac{1}{4}, \frac{\pi}{2}]$ **B** $z = [\frac{1}{2}, \frac{\pi}{4}]$ **A**

$$z = \frac{i}{2\sqrt{3}+2i} = \frac{[\frac{1}{2}, \frac{\pi}{2}]}{4(\frac{2\sqrt{3}}{4} + \frac{2}{4}i)} = \frac{[\frac{1}{2}, \frac{\pi}{2}]}{4(\frac{\sqrt{3}}{2} + \frac{1}{2}i)} = \frac{[\frac{1}{2}, \frac{\pi}{2}]}{[4, \frac{\pi}{6}]} = [\frac{1}{4}, \frac{\pi}{2} - \frac{\pi}{6}] = [\frac{1}{4}, \frac{3\pi}{6} - \frac{\pi}{6}] \Rightarrow z = [\frac{1}{4}, \frac{\pi}{3}]$$

-54 الشكل المثلثي للعدد العقدي : $z = \frac{2}{1+i\sqrt{3}}$

$z = [1, \frac{5\pi}{3}]$ **D** $z = [1, \frac{\pi}{3}]$ **C** $z = [\frac{1}{2}, \frac{\pi}{6}]$ **B** $z = [\frac{1}{2}, \frac{\pi}{3}]$ **A**

$$z = \frac{2}{1+i\sqrt{3}} = \frac{[2, 0]}{2(\frac{1}{2} + \frac{\sqrt{3}}{2}i)} = \frac{[2, 0]}{[2, \frac{\pi}{3}]} = [2, 0 - \frac{\pi}{3}] = [1, 2\pi - \frac{\pi}{3}] \Rightarrow z = [1, \frac{5\pi}{3}]$$

-55 ليكن لدينا الأعداد العقدية التالية : $Z_1 = 1 + i$, $Z_2 = 1 + \sqrt{3}i$ يكون $\arg(Z_1) + \arg(Z_2)$:

$\frac{5\pi}{12}$	D	$\frac{7\pi}{3}$	C	$\frac{5\pi}{3}$	B	$\frac{7\pi}{12}$	A
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$$Z_1 = 1 + i \Rightarrow Z_1 = \sqrt{2} \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i \right) = \left[\sqrt{2}, \frac{\pi}{4} \right], \Rightarrow Z_2 = 1 + \sqrt{3}i \Rightarrow Z_2 = 2 \left(\frac{1}{2} + \frac{\sqrt{3}}{2}i \right) = \left[2, \frac{\pi}{3} \right]$$

$$\arg(Z_1) + \arg(Z_2) = \frac{\pi}{4} + \frac{\pi}{3} = \frac{3\pi}{12} + \frac{4\pi}{12} \Rightarrow \arg(Z_1) + \arg(Z_2) = \frac{7\pi}{12}$$

-56 ليكن لدينا العدد العقدي التالي : $z = \frac{Z_1}{Z_2} = \frac{1+\sqrt{3}i}{\sqrt{3}+i}$ يكون $\arg(Z_1) - \arg(Z_2)$:

$\frac{\pi}{4}$	D	π	C	$\frac{\pi}{6}$	B	$\frac{\pi}{3}$	A
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$$Z_1 = 1 + \sqrt{3}i \Rightarrow Z_1 = 2 \left(\frac{1}{2} + \frac{\sqrt{3}}{2}i \right) = \left[2, \frac{\pi}{3} \right], z_2 = \sqrt{3} + i \Rightarrow z_2 = 2 \left(\frac{\sqrt{3}}{2} + \frac{1}{2}i \right) = \left[2, \frac{\pi}{6} \right]$$

$$\arg(Z_1) - \arg(Z_2) = \frac{\pi}{3} - \frac{\pi}{6} = \frac{2\pi}{6} - \frac{\pi}{6} \Rightarrow \arg(Z_1) - \arg(Z_2) = \frac{\pi}{6}$$

-57 الشكل المثلثي للعدد العقدي : $z = -\sqrt{3} - i$

$z = \left[\sqrt{2}, \frac{\pi}{3} \right]$	D	$z = \left[2, \frac{7\pi}{6} \right]$	C	$\left[2, \frac{\pi}{6} \right]$	B	$z = \left[\sqrt{2}, \frac{7\pi}{6} \right]$	A
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$$z = -\sqrt{3} - i \Rightarrow z = 2 \left(\frac{-\sqrt{3}}{2} - \frac{1}{2}i \right) \Rightarrow z = \left[2, \pi + \frac{\pi}{6} \right] \Rightarrow z = \left[2, \frac{6\pi}{6} + \frac{\pi}{6} \right] \Rightarrow z = \left[2, \frac{7\pi}{6} \right]$$

-58 ليكن لدينا الأعداد العقدية التالية : $Z_1 = 2(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3})$, $Z_2 = 3(\cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3})$ يكون ناتج $Z_1 \cdot Z_2$:

-6	D	6	C	-6i	B	6i	A
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$$Z_1 = \left[2, \frac{\pi}{3} \right], \Rightarrow Z_2 = \left[3, \frac{2\pi}{3} \right] \Rightarrow Z_1 \cdot Z_2 = \left[2 \times 3, \frac{\pi}{3} + \frac{2\pi}{3} \right] = \left[6, \frac{3\pi}{3} \right] = \left[6, \pi \right] \Rightarrow Z_1 \cdot Z_2 = -6$$

-59 يكون ناتج : $\left[2, \frac{\pi}{12} \right] \cdot \left[5, \frac{5\pi}{12} \right]$

-10	D	+10	C	-10i	B	+10i	A
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$$\left[2, \frac{\pi}{12} \right] \cdot \left[5, \frac{5\pi}{12} \right] = \left[10, \frac{\pi}{12} + \frac{5\pi}{12} \right] = \left[10, \frac{6\pi}{12} \right] = \left[10, \frac{\pi}{2} \right] = 10 (\cos \frac{\pi}{2} + i \sin \frac{\pi}{2}) = 10(0 + i) = 10i$$

-60 ليكن لدينا الأعداد العقدية التالية : $Z_1 = 4(\cos \frac{3\pi}{2} + i \sin \frac{3\pi}{2})$, $Z_2 = 2(\cos \frac{4\pi}{3} + i \sin \frac{4\pi}{3})$ يكون ناتج $\frac{Z_1}{Z_2}$:

2	D	2i	C	$\sqrt{3} + i$	B	$1 + \sqrt{3}i$	A
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$$Z_1 = \left[4, \frac{3\pi}{2} \right], \Rightarrow Z_2 = \left[2, \frac{4\pi}{3} \right] \Rightarrow \frac{Z_1}{Z_2} = \left[\frac{4}{2}, \frac{3\pi}{2} - \frac{4\pi}{3} \right] = \left[2, \frac{9\pi}{6} - \frac{8\pi}{6} \right] = \left[2, \frac{\pi}{6} \right] \Rightarrow$$

$$\frac{Z_1}{Z_2} = 2 \left(\cos \frac{\pi}{6} + i \sin \frac{\pi}{6} \right) \Rightarrow \frac{Z_1}{Z_2} = 2 \left(\frac{\sqrt{3}}{2} + \frac{1}{2}i \right) \Rightarrow \frac{Z_1}{Z_2} = \sqrt{3} + i$$

-61 يكون ناتج : $\left[2\sqrt{2}, \frac{7\pi}{6} \right] \div \left[2, \frac{\pi}{6} \right]$

$+\sqrt{2}$	D	$-\sqrt{2}$	C	$-\sqrt{2}i$	B	$+\sqrt{2}i$	A
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$$\left[2\sqrt{2}, \frac{7\pi}{6} \right] \div \left[2, \frac{\pi}{6} \right] = \left[\frac{2\sqrt{2}}{2}, \frac{7\pi}{6} - \frac{\pi}{6} \right] = \left[\sqrt{2}, \frac{6\pi}{6} \right] = \left[\sqrt{2}, \pi \right] = -\sqrt{2}$$

61- يكون ناتج : $[\sqrt{2}, 120^\circ]^2 \cdot [1, 30^\circ]^4$

A $+2i$ B $-2i$ C -2 D 2

$$[\sqrt{2}, 120^\circ]^2 \cdot [1, 30^\circ]^4 = [2, 240^\circ] \cdot [1, 120^\circ] = [2, 240^\circ + 120^\circ] = [2, 360^\circ] = 2$$

62- يكون ناتج : $[6, 140^\circ] \div [2, 50^\circ]$

A $+3i$ B $-3i$ C -3 D 3

$$[6, 140^\circ] \div [2, 50^\circ] = \left[\frac{6}{2}, 140^\circ - 50^\circ\right] = [3, 90^\circ] = 3(\cos 90^\circ + i \sin 90^\circ) = 3(0 + i) = 3i$$

• ليكن لدينا الأعداد العقدية التالية : $Z_1 = 2e^{i\frac{\pi}{4}}$, $Z_2 = \sqrt{3}e^{-i\frac{2\pi}{3}}$, $Z_3 = \frac{3}{2}e^{i\frac{\pi}{2}}$: عن الأسئلة التالية والمطلوب الإجابة عن الأسئلة التالية (من الرقم 63 إلى 68) :

63- يكون العدد العقدي Z_1 بالشكل $z = x + iy$

A $Z_1 = 2 + 2i$ B $Z_1 = \sqrt{2} + \sqrt{2}i$ C $Z_1 = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i$ D $Z_1 = \frac{1}{2} + \frac{1}{2}i$

$$Z_1 = 2e^{i\frac{\pi}{4}} = 2(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4}) = 2\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i\right) = \frac{2}{\sqrt{2}} + \frac{2}{\sqrt{2}}i = \frac{\sqrt{2} \times \sqrt{2}}{\sqrt{2}} + \frac{\sqrt{2} \times \sqrt{2}}{\sqrt{2}}i \Rightarrow Z_1 = \sqrt{2} + \sqrt{2}i$$

64- يكون العدد العقدي Z_2 بالشكل $z = x + iy$

A $Z_2 = \frac{\sqrt{3}}{2} + \frac{3}{2}i$ B $Z_2 = -\frac{3}{2} - \frac{\sqrt{3}}{2}i$ C $Z_2 = -\frac{\sqrt{3}}{2} - \frac{3}{2}i$ D $Z_2 = -\frac{3}{2} + \frac{\sqrt{3}}{2}i$

$$Z_2 = \sqrt{3}e^{-i\frac{2\pi}{3}} = \sqrt{3}(\cos(-\frac{2\pi}{3}) + i \sin(-\frac{2\pi}{3})) = \sqrt{3}\left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right) \Rightarrow Z_2 = -\frac{\sqrt{3}}{2} - \frac{3}{2}i$$

65- يكون العدد العقدي Z_3 بالشكل $z = x + iy$

A $Z_3 = -\frac{3}{2}i$ B $Z_3 = \frac{3}{2}$ C $Z_3 = -\frac{3}{2}$ D $Z_3 = \frac{3}{2}i$

$$Z_3 = \frac{3}{2}e^{i\frac{\pi}{2}} = \left[\frac{3}{2}, \frac{\pi}{2}\right] \Rightarrow Z_3 = \frac{3}{2}i$$

66- يكون العدد العقدي $Z_1 \cdot Z_3$ بالشكل الأسّي يساوي :

A $3e^{i(\frac{3\pi}{4})}$ B $3e^{i(\frac{\pi}{2})}$ C $3e^{i(\frac{3\pi}{2})}$ D $3e^{i(\frac{\pi}{4})}$

$$Z_1 \cdot Z_3 = 2e^{i\frac{\pi}{4}} \times \frac{3}{2}e^{i\frac{\pi}{2}} = 3e^{i(\frac{\pi}{4} + \frac{\pi}{2})} = 3e^{i(\frac{\pi}{4} + \frac{2\pi}{4})} \Rightarrow Z_1 \cdot Z_3 = 3e^{i(\frac{3\pi}{4})}$$

67- يكون العدد العقدي $\frac{Z_1}{Z_3}$ بالشكل الأسّي يساوي :

A $\frac{4}{3}e^{i(\frac{7\pi}{4})}$ B $\frac{4}{3}e^{i(\frac{\pi}{4})}$ C $\frac{3}{4}e^{i(\frac{7\pi}{4})}$ D $\frac{3}{4}e^{i(\frac{\pi}{4})}$

$$\frac{Z_1}{Z_3} = 2e^{i\frac{\pi}{4}} \div \frac{3}{2}e^{i\frac{\pi}{2}} = \frac{2}{\frac{3}{2}}e^{i(\frac{\pi}{4} - \frac{\pi}{2})} = 2 \times \frac{2}{3}e^{i(\frac{\pi}{4} - \frac{2\pi}{4})} = \frac{4}{3}e^{i(-\frac{\pi}{4})} = \frac{4}{3}e^{i(2\pi - \frac{\pi}{4})} \Rightarrow \frac{Z_1}{Z_3} = \frac{4}{3}e^{i(\frac{7\pi}{4})}$$

68- يكون العدد العقدي Z_2^5 بالشكل الأسّي يساوي :

A $9e^{i\frac{2\pi}{3}}$ B $9\sqrt{3}e^{i\frac{4\pi}{3}}$ C $9\sqrt{3}e^{i\frac{2\pi}{3}}$ D $9e^{i\frac{4\pi}{3}}$

$$Z_2 = \sqrt{3}e^{-i\frac{2\pi}{3}} \Rightarrow Z_2^5 = (\sqrt{3})^5 e^{-i(\frac{2\pi}{3}) \times 5} = 9\sqrt{3}e^{-i(\frac{10\pi}{3})} = 9\sqrt{3}e^{i(2\pi - \frac{10\pi}{3})} = 9\sqrt{3}e^{i(-\frac{4\pi}{3})} = 9\sqrt{3}e^{i(2\pi - \frac{4\pi}{3})} \Rightarrow Z_2^5 = 9\sqrt{3}e^{i\frac{2\pi}{3}}$$

- ليكن لدينا الأعداد العقدية التالية : $Z_1 = e^{i\frac{\pi}{3}}$, $Z_2 = 3e^{-i\frac{\pi}{4}}$, $Z_3 = \sqrt{2}e^{i\frac{2\pi}{3}}$ عن الأسئلة التالية والمطلوب الإجابة عن الأسئلة التالية (من الرقم 69 إلى 73) :

69- يكون العدد العقدي $Z_1 \cdot Z_2$ بالشكل الأسّي يساوي :

$3e^{i(\frac{\pi}{12})}$	D	$3e^{i(\frac{\pi}{6})}$	C	$3e^{i(\frac{\pi}{3})}$	B	$3e^{i(\frac{\pi}{4})}$	A
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$$Z_1 \cdot Z_2 = e^{i\frac{\pi}{3}} \cdot 3e^{-i\frac{\pi}{4}} = 3e^{i(\frac{\pi}{3} - \frac{\pi}{4})} = 3e^{i(\frac{4\pi}{12} - \frac{3\pi}{12})} \Rightarrow Z_1 \cdot Z_2 = 3e^{i(\frac{\pi}{12})}$$

70- يكون العدد العقدي $\frac{Z_1}{Z_2}$ بالشكل الأسّي يساوي :

$3e^{i(\frac{\pi}{12})}$	D	$\frac{1}{3}e^{i(\frac{\pi}{12})}$	C	$3e^{i(\frac{7\pi}{12})}$	B	$\frac{1}{3}e^{i(\frac{7\pi}{12})}$	A
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$$\frac{Z_1}{Z_2} = e^{i\frac{\pi}{3}} \div 3e^{-i\frac{\pi}{4}} = \frac{1}{3}e^{i(\frac{\pi}{3} - (-\frac{\pi}{4}))} = \frac{1}{3}e^{i(\frac{4\pi}{12} + \frac{3\pi}{12})} \Rightarrow \frac{Z_1}{Z_2} = \frac{1}{3}e^{i(\frac{7\pi}{12})}$$

71- يكون العدد العقدي Z_1^3 بالشكل الديكارتي يساوي :

$Z_1^3 = -i$	D	$Z_1^3 = i$	C	$Z_1^3 = -1$	B	$Z_1^3 = 1$	A
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$$Z_1 = e^{i\frac{\pi}{3}} \Rightarrow Z_1^3 = 1^3 e^{i(\frac{\pi}{3} \times 3)} \Rightarrow Z_1^3 = 1e^{i(\pi)} \Rightarrow Z_1^3 = [1, \pi] \Rightarrow Z_1^3 = -1$$

72- يكون العدد العقدي $Z_1 \cdot Z_2 \cdot Z_3$ بالشكل الأسّي يساوي :

$3\sqrt{2}e^{i(\frac{\pi}{4})}$	D	$3\sqrt{2}e^{i(\frac{7\pi}{12})}$	C	$3\sqrt{2}e^{i(\frac{\pi}{12})}$	B	$3\sqrt{2}e^{i(\frac{3\pi}{4})}$	A
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$$Z_1 \cdot Z_2 \cdot Z_3 = e^{i\frac{\pi}{3}} \cdot 3e^{-i\frac{\pi}{4}} \cdot \sqrt{2}e^{i\frac{2\pi}{3}} = 3\sqrt{2}e^{i(\frac{\pi}{3} + \frac{2\pi}{3} - \frac{\pi}{4})} = 3\sqrt{2}e^{i(\pi - \frac{\pi}{4})} \Rightarrow Z_1 \cdot Z_2 \cdot Z_3 = 3\sqrt{2}e^{i(\frac{3\pi}{4})}$$

73- يكون العدد العقدي $\frac{Z_2}{Z_3}$ بالشكل الأسّي يساوي :

$\frac{3}{\sqrt{2}}e^{i(\frac{\pi}{4})}$	D	$\frac{3}{\sqrt{2}}e^{i(\frac{5\pi}{12})}$	C	$\frac{3}{\sqrt{2}}e^{i(\frac{13\pi}{12})}$	B	$\frac{3}{\sqrt{2}}e^{i(\frac{7\pi}{12})}$	A
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$$\frac{Z_2}{Z_3} = 3e^{-i\frac{\pi}{4}} \div \sqrt{2}e^{i\frac{2\pi}{3}} = \frac{3}{\sqrt{2}}e^{i(-\frac{\pi}{4} - \frac{2\pi}{3})} = \frac{3}{\sqrt{2}}e^{i(-\frac{3\pi}{12} - \frac{8\pi}{12})} = \frac{3}{\sqrt{2}}e^{i(-\frac{11\pi}{12})}$$

$$\frac{Z_2}{Z_3} = \frac{3}{\sqrt{2}}e^{i(2\pi - \frac{11\pi}{12})} \Rightarrow \frac{Z_2}{Z_3} = \frac{3}{\sqrt{2}}e^{i(\frac{13\pi}{12})}$$

74- يكون العدد العقدي $z = 3ie^{i\frac{\pi}{3}}$ بالشكل الأسّي يساوي :

$z = 3e^{i\frac{4\pi}{3}}$	D	$z = 3e^{i\frac{5\pi}{6}}$	C	$z = 3e^{i\frac{\pi}{6}}$	B	$z = 3e^{i\frac{\pi}{3}}$	A
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$$z = 3ie^{i\frac{\pi}{3}} \Rightarrow z = 3e^{i\frac{\pi}{2}}e^{i\frac{\pi}{3}} \Rightarrow z = 3e^{i\frac{5\pi}{6}}$$

75- يكون العدد العقدي $z = 2\sqrt{3} + 6i$ بالشكل الأسّي يساوي :

$z = 4e^{i\frac{\pi}{3}}$	D	$z = 4\sqrt{3}e^{i\frac{\pi}{3}}$	C	$z = 4\sqrt{3}e^{i\frac{\pi}{4}}$	B	$z = 4\sqrt{3}e^{i\frac{\pi}{6}}$	A
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$$z = 2\sqrt{3} + 6i \Rightarrow r = \sqrt{(2\sqrt{3})^2 + 6^2} \Rightarrow r = \sqrt{12 + 36} \Rightarrow r = \sqrt{48} \Rightarrow r = \sqrt{16 \times 3} \Rightarrow r = 4\sqrt{3}$$

$$\cos(\theta) = \frac{2\sqrt{3}}{4\sqrt{3}} = \frac{1}{2}, \sin(\theta) = \frac{6}{4\sqrt{3}} = \frac{3}{2\sqrt{3}} = \frac{\sqrt{3} \times \sqrt{3}}{2\sqrt{3}} = \frac{\sqrt{3}}{2} \Rightarrow \theta = \frac{\pi}{3} \Rightarrow z = 4\sqrt{3}e^{i\frac{\pi}{3}}$$

76- يكون العدد العقدي $z = (1 + \sqrt{3}i)^4$ بالشكل الأسّي يساوي :

$z = 16 e^{i\frac{4\pi}{3}}$	D	$z = 16 e^{i\frac{2\pi}{3}}$	C	$z = 16 e^{i\frac{4\pi}{6}}$	B	$z = 16 e^{i\frac{\pi}{3}}$	A
------------------------------	---	------------------------------	---	------------------------------	---	-----------------------------	---

$$z = (1 + \sqrt{3}i)^4 = [2(\frac{1}{2} + \frac{\sqrt{3}}{2}i)]^4 = [2, \frac{\pi}{3}]^4 = [16, \frac{4\pi}{3}] \Rightarrow z = 16 e^{i\frac{4\pi}{3}}$$

77- يكون العدد العقدي $z = \frac{1+i}{\sqrt{3}+i}$ بالشكل الأسّي يساوي :

$z = \frac{1}{\sqrt{2}} e^{i\frac{4\pi}{3}}$	D	$z = \frac{1}{2} e^{i\frac{2\pi}{3}}$	C	$z = \frac{1}{2} e^{i\frac{4\pi}{3}}$	B	$z = \frac{1}{\sqrt{2}} e^{i\frac{\pi}{12}}$	A
--	---	---------------------------------------	---	---------------------------------------	---	--	---

$$z = \frac{1+i}{\sqrt{3}+i} = \frac{[\sqrt{2}, \frac{\pi}{4}]}{[2, \frac{\pi}{6}]} = [\frac{\sqrt{2}}{2}, \frac{\pi}{4} - \frac{\pi}{6}] = [\frac{1}{\sqrt{2}}, \frac{3\pi}{12} - \frac{2\pi}{12}] = 2[\frac{1}{\sqrt{2}}, \frac{\pi}{12}] = [\frac{1}{\sqrt{2}}, \frac{\pi}{12}] \Rightarrow z = \frac{1}{\sqrt{2}} e^{i\frac{\pi}{12}}$$

78- يكون العدد العقدي $z = \frac{6}{1+i}$ بالشكل الأسّي يساوي :

$z = 6\sqrt{2} e^{i\frac{4\pi}{3}}$	D	$z = 6\sqrt{2} e^{i\frac{2\pi}{3}}$	C	$z = 3\sqrt{2} e^{i\frac{7\pi}{4}}$	B	$z = 3\sqrt{2} e^{i\frac{\pi}{12}}$	A
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$$= \frac{6}{1+i} = \frac{[6, 0]}{[\sqrt{2}, \frac{\pi}{4}]} = [\frac{6}{\sqrt{2}}, 0 - \frac{\pi}{4}] = [\frac{2 \times 3}{\sqrt{2}}, -\frac{\pi}{4}] = [3\sqrt{2}, 2\pi - \frac{\pi}{4}] = [\frac{1}{\sqrt{2}}, \frac{7\pi}{4}] \Rightarrow z = 3\sqrt{2} e^{i\frac{7\pi}{4}}$$

79- يكون العدد العقدي $z = -5e^{i\frac{\pi}{4}}$ بالشكل الأسّي يساوي :

$z = \sqrt{5} e^{i\frac{5\pi}{6}}$	D	$z = 5 e^{i\frac{4\pi}{3}}$	C	$z = 5 e^{i\frac{\pi}{3}}$	B	$z = \sqrt{5} e^{i\frac{4\pi}{3}}$	A
------------------------------------	---	-----------------------------	---	----------------------------	---	------------------------------------	---

$$z = -5e^{i\frac{\pi}{4}} \Rightarrow z = 5 e^{i\pi} e^{i\frac{\pi}{3}} \Rightarrow z = 5 e^{i\frac{4\pi}{3}}$$

80- يكون العدد العقدي $z = \frac{3}{2} + \frac{\sqrt{3}}{2}i$ بالشكل الأسّي يساوي :

$z = \sqrt{3} e^{i\frac{\pi}{6}}$	D	$z = 3 e^{i\frac{\pi}{6}}$	C	$z = \sqrt{3} e^{i\frac{\pi}{3}}$	B	$z = 3 e^{i\frac{\pi}{3}}$	A
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$$z = \frac{3}{2} + \frac{\sqrt{3}}{2}i \Rightarrow z = \sqrt{3} (\frac{\sqrt{3}}{2} + \frac{1}{2}i) \Rightarrow z = \sqrt{3} e^{i\frac{\pi}{6}}$$

81- يكون العدد العقدي $z = \frac{\sqrt{3}}{2} e^{-i\frac{\pi}{6}}$ بالشكل الديكارتي يساوي :

$z = \frac{\sqrt{3}}{4} - i\frac{3}{4}$	D	$z = \frac{\sqrt{3}}{4} + i\frac{3}{4}$	C	$z = \frac{3}{4} + i\frac{\sqrt{3}}{4}$	B	$z = \frac{3}{4} - i\frac{\sqrt{3}}{4}$	A
---	---	---	---	---	---	---	---

$$z = \frac{\sqrt{3}}{2} e^{-i\frac{\pi}{6}} \Rightarrow z = \frac{\sqrt{3}}{2} (\cos(-\frac{\pi}{6}) + i \sin(-\frac{\pi}{6})) \Rightarrow z = \frac{\sqrt{3}}{2} (\frac{\sqrt{3}}{2} - i\frac{1}{2}) \Rightarrow z = \frac{3}{4} - i\frac{\sqrt{3}}{4}$$

82- يكون العدد العقدي $z = \frac{1+i+i^2+i^3+i^4}{1-i}$ بالشكل الديكارتي يساوي :

$z = \frac{\sqrt{3}}{4} - i\frac{3}{4}$	D	$z = \frac{\sqrt{3}}{4} + i\frac{3}{4}$	C	$z = \frac{1}{2} + i\frac{1}{2}$	B	$z = \frac{3}{4} - i\frac{\sqrt{3}}{4}$	A
---	---	---	---	----------------------------------	---	---	---

$$z = \frac{1+i+i^2+i^3+i^4}{1-i} = \frac{1+i-1-i+1}{1-i} = \frac{1}{1-i} = \frac{1(1+i)}{(1-i)(1+i)} = \frac{1+i}{1+1} = \frac{1+i}{2} \Rightarrow z = \frac{1}{2} + i\frac{1}{2}$$

83- ليكون لدينا المعادلة التالية: $z^2 + 6z + 34 = 0$ يكون أحد حلولها $z_1 = -3 - 5i$ يكون الحل الآخر هو :

$z_2 = -3 - 5i$	D	$z_2 = -3 + 5i$	C	$z_2 = +3 - 5i$	B	$z_2 = +3 + 5i$	A
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$$z_1 = -3 - 5i \Rightarrow z_2 = -3 + 5i$$

-84 ليكون لدينا المعادلة التالية: $2z^2 - 6z + 5 = 0$ تكون حلول المعادلة :

A $(1+i), (1-i)$ **B** $(\frac{3}{4}+\frac{1}{4}i), (\frac{3}{4}-\frac{1}{4}i)$ **C** $(\frac{3}{2}+\frac{1}{2}i), (-\frac{3}{2}+\frac{1}{2}i)$ **D** $(\frac{3}{2}+\frac{1}{2}i), (\frac{3}{2}-\frac{1}{2}i)$

$$\Delta = 36 - 4(2)(5) = 36 - 40 = -4 = 4i^2 \Rightarrow \sqrt{\Delta} = 2i \Rightarrow z_1 = \frac{6+2i}{4} \Rightarrow z_1 = \frac{3}{2} + \frac{1}{2}i \Rightarrow z_2 = \frac{3}{2} - \frac{1}{2}i$$

-85 ليكون لدينا المعادلة التالية: $z^2 - 5z + 9 = 0$ تكون حلول المعادلة :

A $(\frac{5}{2}+\frac{3}{2}i), (\frac{5}{2}-\frac{3}{2}i)$ **B** $(\frac{5}{2}+\frac{9}{2}i), (\frac{5}{2}-\frac{9}{2}i)$ **C** $(\frac{5}{2}+\frac{3}{2}i), (-\frac{5}{2}-\frac{3}{2}i)$ **D** $(\frac{5}{2}+\frac{3}{2}i), (-\frac{5}{2}+\frac{3}{2}i)$

$$\Delta = 25 - 4(1)(9) = 25 - 36 = -9 = 9i^2 \Rightarrow \sqrt{\Delta} = 3i \Rightarrow z_1 = \frac{5+3i}{2} \Rightarrow z_1 = \frac{5}{2} + \frac{3}{2}i \Rightarrow z_2 = \frac{5}{2} - \frac{3}{2}i$$

-86 ليكون لدينا المعادلة التالية: $z^2 - 2z + 3 = 0$ تكون حلول المعادلة :

A $(1+\frac{3}{2}i), (-1-\frac{3}{2}i)$ **B** $(1+\frac{9}{2}i), (-1+\frac{9}{2}i)$ **C** $(\frac{5}{2}+\frac{3}{2}i), (-1+\frac{3}{2}i)$ **D** $(1+\frac{3}{2}i), (1-\frac{3}{2}i)$

$$\Delta = 4 - 4(1)(3) = 4 - 12 = -8 = 8i^2 \Rightarrow \sqrt{\Delta} = 2\sqrt{2}i \Rightarrow z_1 = \frac{2+2\sqrt{2}i}{2} \Rightarrow z_1 = 1 + \sqrt{2}i \Rightarrow z_2 = 1 - \sqrt{2}i$$

-87 ليكون لدينا المعادلة التالية: $z^2 + z + 1 = 0$ تكون حلول المعادلة :

A $(-\frac{1}{2}+\frac{\sqrt{3}}{2}i), (-\frac{1}{2}-\frac{\sqrt{3}}{2}i)$ **B** $(\frac{\sqrt{3}}{2}+i\frac{1}{2}), (\frac{\sqrt{3}}{2}-i\frac{1}{2})$ **C** $(\frac{1}{2}+\frac{\sqrt{3}}{2}i), (-\frac{1}{2}-\frac{\sqrt{3}}{2}i)$ **D** $(\frac{1}{2}+\frac{\sqrt{3}}{2}i), (-\frac{1}{2}+\frac{\sqrt{3}}{2}i)$

$$\Delta = 1 - 4(1)(1) = 1 - 4 = -3 = 3i^2 \Rightarrow \sqrt{\Delta} = \sqrt{3}i \Rightarrow z_1 = \frac{-1+\sqrt{3}i}{2} \Rightarrow z_1 = -\frac{1}{2} + \frac{\sqrt{3}}{2}i \Rightarrow z_2 = -\frac{1}{2} - \frac{\sqrt{3}}{2}i$$

-88 ليكون لدينا المعادلة التالية: $z^2 + 4z + 5 = 0$ تكون حلول المعادلة :

A $(2-2i), (2+2i)$ **B** $(-2-i), (-2+i)$ **C** $(-2-i), (-2-i)$ **D** $(+2-2i), (-2-2i)$

$$\Delta = 16 - 4(1)(5) = 16 - 20 = -4 = 4i^2 \Rightarrow \sqrt{\Delta} = 2i \Rightarrow z_1 = \frac{-4+2i}{2} \Rightarrow z_1 = -2 + i \Rightarrow z_2 = -2 - i$$

-89 ليكون لدينا المعادلة التالية: $z^2 + 4z + 4 = 0$ تكون حلول المعادلة :

A 2 **B** $-2i$ **C** $-2+i$ **D** -2

$$\Delta = 16 - 4(1)(4) = 16 - 16 = 0 \Rightarrow \sqrt{\Delta} = 0 \Rightarrow z_1 = \frac{-4}{2} \Rightarrow z_1 = z_2 = -2$$

-90 ليكون لدينا المعادلة التالية: $z^2 + 6z + 34 = 0$ يكون مجموع الحلول لها :

A $z_1 + z_2 = 6$ **B** $z_1 + z_2 = -6$ **C** $z_1 + z_2 = 34$ **D** $z_1 + z_2 = -34$

$$z_1 + z_2 = \frac{-b}{a} \Rightarrow z_1 + z_2 = \frac{-6}{1} \Rightarrow z_1 + z_2 = -6$$

-91 ليكون لدينا المعادلة التالية: $z^2 + 6z + 34 = 0$ يكون جداء الحلول لها :

A $z_1 \cdot z_2 = 6$ **B** $z_1 \cdot z_2 = -6$ **C** $z_1 \cdot z_2 = 34$ **D** $z_1 \cdot z_2 = -34$

$$z_1 \cdot z_2 = \frac{c}{a} \Rightarrow z_1 \cdot z_2 = \frac{34}{1} \Rightarrow z_1 \cdot z_2 = 34$$

92- ليكون لدينا المعادلة التالية: $2z^2 - 6z + 5 = 0$ يكون مجموع حلول المعادلة :

$$z_1 + z_2 = 3$$

D

$$z_1 + z_2 = 6i$$

C

$$z_1 + z_2 = -6$$

B

$$z_1 + z_2 = -3$$

A

$$z_1 + z_2 = \frac{-b}{a} \Rightarrow z_1 + z_2 = \frac{6}{2} \Rightarrow z_1 + z_2 = 3$$

93- ليكون لدينا المعادلة التالية: $2z^2 - 6z + 5 = 0$ يكون جداء حلول المعادلة :

$$z_1 \cdot z_2 = -3$$

D

$$z_1 \cdot z_2 = 3$$

C

$$z_1 \cdot z_2 = \frac{-5}{2}$$

B

$$z_1 \cdot z_2 = \frac{5}{2}$$

A

$$z_1 \cdot z_2 = \frac{c}{a} \Rightarrow z_1 \cdot z_2 = \frac{5}{2}$$

93- ليكون لدينا المعادلة التالية: $z^2 + bz + c = 0$ عين قيمة كل من c و b لتكون الحلول لهذه المعادلة هي :

$$: z_1 = -\frac{1}{2} + \frac{\sqrt{3}}{2}i \text{ و } z_2 = -\frac{1}{2} - \frac{\sqrt{3}}{2}i$$

$$b = 1, c = -1$$

D

$$b = -1, c = -1$$

C

$$b = 1, c = 1$$

B

$$b = -1, c = 1$$

A

$$z_1 + z_2 = \frac{-b}{a} \Rightarrow -\frac{1}{2} + \frac{\sqrt{3}}{2}i - \frac{1}{2} - \frac{\sqrt{3}}{2}i = \frac{-b}{1} \Rightarrow -\frac{2}{2} = \frac{-b}{1} \Rightarrow -b = \frac{-2 \times 1}{2} \Rightarrow b = 1$$

$$z_1 \cdot z_2 = \frac{c}{a} \Rightarrow \left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) \left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right) = \frac{c}{1} \Rightarrow \frac{1}{4} + \frac{3}{4} = \frac{c}{1} \Rightarrow \frac{4}{4} = \frac{c}{1} \Rightarrow c = \frac{4 \times 1}{4} \Rightarrow c = 1$$

94- ليكون لدينا المعادلة التالية: $2z^2 + bz + c = 0$ عين قيمة كل من c و b لتكون الحلول لهذه المعادلة هي :

$$: z_1 = \frac{3}{2} + \frac{1}{2}i \text{ و } z_2 = \frac{3}{2} - \frac{1}{2}i$$

$$b = -5, c = -6$$

D

$$b = -6, c = 5$$

C

$$b = 6, c = -5$$

B

$$b = 6, c = 5$$

A

$$z_1 + z_2 = \frac{-b}{a} \Rightarrow \frac{3}{2} + \frac{1}{2}i + \frac{3}{2} - \frac{1}{2}i = \frac{-b}{2} \Rightarrow \frac{6}{2} = \frac{-b}{2} \Rightarrow -b = \frac{6 \times 2}{2} \Rightarrow b = -6$$

$$z_1 \cdot z_2 = \frac{c}{a} \Rightarrow \left(\frac{3}{2} + \frac{1}{2}i\right) \left(\frac{3}{2} - \frac{1}{2}i\right) = \frac{c}{2} \Rightarrow \frac{9}{4} + \frac{1}{4} = \frac{c}{2} \Rightarrow \frac{10}{4} = \frac{c}{2} \Rightarrow c = \frac{10 \times 2}{4} \Rightarrow c = 5$$

95- ليكن لدينا الأعداد العقدية التالية: $Z_1 = 2 + 2i$, $Z_2 = -4 - 2i$ يكون $\arg(Z_1 + Z_2)$:

$$\pi$$

D

$$\frac{3\pi}{2}$$

C

$$\frac{\pi}{2}$$

B

$$0$$

A

$$\arg(Z_1 + Z_2) = \arg(2 + 2i - 4 - 2i) = \arg(-2) \Rightarrow \arg(Z_1 + Z_2) = \pi$$

• ليكن لدينا الأعداد العقدية التالية: $Z_C = \sqrt{3} + i$, $Z_A = 1 + i$ والمطلوب الإجابة عن الأسئلة التالية (من

سؤال دورة للاختصاصات الكهربائية الدورة الأولى 2024

الرقم 96 إلى 100) :

96- يكون ناتج ما يلي: Z_A^5 بالشكل الديكارتي :

$$Z_A^5 = 2 - 2i$$

D

$$Z_A^5 = 2\sqrt{2} + 2\sqrt{2}i$$

C

$$Z_A^5 = 4 + 4i$$

B

$$Z_A^5 = -4 - 4i$$

A

$$Z_A = 1 + i \Rightarrow Z_A = \sqrt{2} \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i \right) \Rightarrow Z_A = [\sqrt{2}, \frac{\pi}{4}] \Rightarrow Z_A^5 = [4\sqrt{2}, \frac{5\pi}{4}] \Rightarrow Z_A^5 = 4\sqrt{2}(\cos \frac{5\pi}{4} + i \sin \frac{5\pi}{4})$$

$$\Rightarrow Z_A^5 = 4\sqrt{2} \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i \right) \Rightarrow Z_A^5 = \left(-\frac{4\sqrt{2}}{\sqrt{2}} - \frac{4\sqrt{2}}{\sqrt{2}}i \right) \Rightarrow Z_A^5 = -4 - 4i$$

97- يكون Z_A بالشكل الأسّي:

$Z_A = \frac{2}{\sqrt{2}} e^{i\frac{\pi}{4}}$	D	$Z_A = 2e^{i\frac{\pi}{6}}$	C	$Z_A = \sqrt{2}e^{i\frac{\pi}{4}}$	B	$Z_A = \sqrt{2}e^{i\frac{\pi}{3}}$	A
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$$Z_A = 1 + i \Rightarrow Z_A = \sqrt{2} \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i \right) \Rightarrow Z_A = [\sqrt{2}, \frac{\pi}{4}] \Rightarrow Z_A = \sqrt{2}e^{i\frac{\pi}{4}}$$

98- يكون Z_C بالشكل الأسّي:

$Z_A = \frac{2}{\sqrt{2}} e^{i\frac{\pi}{4}}$	D	$Z_A = 2e^{i\frac{\pi}{6}}$	C	$Z_A = \sqrt{2}e^{i\frac{\pi}{4}}$	B	$Z_A = \sqrt{2}e^{i\frac{\pi}{3}}$	A
---	---	-----------------------------	---	------------------------------------	---	------------------------------------	---

$$Z_C = \sqrt{3} + i \Rightarrow Z_C = 2 \left(\frac{\sqrt{3}}{2} + \frac{1}{2}i \right) \Rightarrow Z_C = [2, \frac{\pi}{6}] \Rightarrow Z_C = 2e^{i\frac{\pi}{6}}$$

99- يكون $\frac{Z_A}{Z_C}$ بالشكل الأسّي:

$z = \frac{1}{\sqrt{2}} e^{i\frac{4\pi}{3}}$	D	$z = \frac{1}{2} e^{i\frac{2\pi}{3}}$	C	$z = \frac{1}{2} e^{i\frac{4\pi}{3}}$	B	$z = \frac{1}{\sqrt{2}} e^{i\frac{\pi}{12}}$	A
--	---	---------------------------------------	---	---------------------------------------	---	--	---

$$\frac{Z_A}{Z_C} = \frac{[\sqrt{2}, \frac{\pi}{4}]}{[2, \frac{\pi}{6}]} \Rightarrow \frac{Z_A}{Z_C} = \left[\frac{\sqrt{2}}{2}, \frac{\pi}{4} - \frac{\pi}{6} \right] = \left[\frac{1}{\sqrt{2}}, \frac{3\pi}{12} - \frac{2\pi}{12} \right] = \left[\frac{1}{\sqrt{2}}, \frac{\pi}{12} \right] \Rightarrow \frac{Z_A}{Z_C} = \frac{1}{\sqrt{2}} e^{i\frac{\pi}{12}}$$

100- يكون ناتج $\arg(Z_A - Z_C)$:

π	D	$\frac{3\pi}{2}$	C	$\frac{\pi}{2}$	B	0	A
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$$\arg(Z_A - Z_C) = \arg(1 + i - \sqrt{3} - i) = \arg(1 - \sqrt{3}) \Rightarrow \arg(-(\sqrt{3} - 1)) = \pi$$

• ليكن لدينا الأعداد العقدية التالية: $Z_A = 2 - 2\sqrt{3}i$ والمطلوب الإجابة عن الأسئلة التالية (من الرقم 101 إلى 103):

سؤال بكرة للاختصاصات الميكانيكة الدورة الأولى والكهربائية الدورة الثانية 2024

101- يكون المرافق للعدد العقدي $\overline{Z_A}$ هو:

$\overline{Z_A} = 2\sqrt{3} - 2i$	D	$\overline{Z_A} = -2 + 2\sqrt{3}i$	C	$\overline{Z_A} = -2 - 2\sqrt{3}i$	B	$\overline{Z_A} = 2 + 2\sqrt{3}i$	A
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$$Z_A = 2 - 2\sqrt{3}i \Rightarrow \overline{Z_A} = 2 + 2\sqrt{3}i$$

102- تكون طويلة العدد العقدي $|Z_A|$ هو:

$ Z_A = \sqrt{8}$	D	$ Z_A = 2$	C	$ Z_A = 4$	B	$ Z_A = 16$	A
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$$|Z_A| = \sqrt{2^2 + (2\sqrt{3})^2} \Rightarrow |Z_A| = \sqrt{4 + 12} \Rightarrow |Z_A| = \sqrt{16} \Rightarrow |Z_A| = 4$$

103- يكون Z_A بالشكل الأسّي:

$Z_A = 2e^{i\frac{\pi}{3}}$	D	$Z_A = 4e^{i\frac{5\pi}{3}}$	C	$Z_A = 2e^{i\frac{5\pi}{6}}$	B	$Z_A = 4e^{i\frac{\pi}{3}}$	A
-----------------------------	---	------------------------------	---	------------------------------	---	-----------------------------	---

$$Z_A = 2 - 2\sqrt{3}i \Rightarrow Z_A = 4 \left(\frac{1}{2} - \frac{\sqrt{3}}{2}i \right) \Rightarrow Z_A = [4, 2\pi - \frac{\pi}{3}] \Rightarrow Z_A = [4, \frac{5\pi}{3}] \Rightarrow Z_A = 4e^{i\frac{5\pi}{3}}$$

- ليكن لدينا الأعداد العقدية التالية: $Z_A = 2 + 2\sqrt{3}i$ والمطلوب الإجابة عن الأسئلة التالية (من الرقم 103 إلى 106): سؤال دورة للاختصاصات الميكاتريكة الدورة الثانية 2024

106- يكون المرافق للعدد العقدي $\overline{Z_A}$ هو :

$\overline{Z_A} = 2\sqrt{3} - 2i$ D $\overline{Z_A} = -2 + 2\sqrt{3}i$ C $\overline{Z_A} = -2 - 2\sqrt{3}i$ B $\overline{Z_A} = 2 - 2\sqrt{3}i$ A

$$Z_A = 2 + 2\sqrt{3}i \Rightarrow \overline{Z_A} = 2 - 2\sqrt{3}i$$

107- تكون طويلة العدد العقدي $|Z_A|$ هو :

$|Z_A| = \sqrt{8}$ D $|Z_A| = 2$ C $|Z_A| = 4$ B $|Z_A| = 16$ A

$$|Z_A| = \sqrt{2^2 + (2\sqrt{3})^2} \Rightarrow |Z_A| = \sqrt{4 + 12} \Rightarrow |Z_A| = \sqrt{16} \Rightarrow |Z_A| = 4$$

108- يكون Z_A بالشكل الأسّي:

$Z_A = 2e^{i\frac{\pi}{3}}$ D $Z_A = 4e^{i\frac{5\pi}{6}}$ C $Z_A = 2e^{i\frac{5\pi}{6}}$ B $Z_A = 4e^{i\frac{\pi}{3}}$ A

$$Z_A = 2 + 2\sqrt{3}i \Rightarrow Z_A = 4\left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) \Rightarrow Z_A = [4, \frac{\pi}{3}] \Rightarrow Z_A = [4, \frac{\pi}{3}] \Rightarrow Z_A = 4e^{i\frac{\pi}{3}}$$

109- يكون ناتج ما يلي: Z_A^2 بالشكل الأسّي :

$Z_A^2 = 4e^{i\frac{2\pi}{3}}$ D $Z_A^2 = 16e^{i\frac{5\pi}{3}}$ C $Z_A^2 = 16e^{i\frac{2\pi}{3}}$ B $Z_A^2 = 4e^{i\frac{5\pi}{3}}$ A

$$Z_A^2 = 4^2 e^{i2 \times \frac{\pi}{3}} \Rightarrow Z_A^2 = 16 e^{i\frac{2\pi}{3}}$$

SAMER ALSOSO
ELECTRONICS ENGINEER

ملحق تصحيح بنك العقديّة

التصويب	الخطأ	رقم السؤال
$2 Z_1 - 2 \overline{Z_2}$ تغير العبارة في نص السؤال وفي الحل	$2 Z_1 - 3 \overline{Z_2}$	12
الخيار A	الخيار C	45
$[3\sqrt{2}, \frac{7\pi}{4}]$ في الحل	$[\frac{1}{\sqrt{2}}, \frac{7\pi}{4}]$ في الحل	78
في السؤال $-5e^{i\frac{\pi}{3}}$	في السؤال $-5e^{i\frac{\pi}{4}}$	79
حذف الأسئلة بالكامل		86+85
الخيار الصحيح A	يوجد خياران A و D	93
في الحل في السطر الثاني $-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i$	في الحل في السطر الثاني $\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i$	96
الخيار الصحيح A $\frac{z_A}{z_C} = \frac{[\sqrt{2}, \frac{\pi}{4}]}{[2, \frac{\pi}{6}]}$ في الحل	يوجد خياران A و C $\frac{z_A}{z_C} = \frac{[\sqrt{2}, \frac{\pi}{4}]}{[2, \frac{\pi}{4}]}$ في الحل	99